Non-Gaussianity of the density distribution in accelerating universes

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Abstract. According to recent observations, the existence of the dark energy has been considered. Even though we have obtained the constraint of the equation of the state for dark energy ($p = w\rho$) as $-1 \leq w \leq -0.78$ by combining WMAP data with other astronomical data, in order to pin down w, it is necessary to use other independent observational tools. For this purpose, we consider the w dependence of the non-Gaussianity of the density distribution generated by nonlinear dynamics. To extract the non-Gaussianity, we follow a semi-analytic approach based on Lagrangian linear perturbation theory, which provides an accurate value for the quasi-nonlinear region. From our results, the difference of the non-Gaussianity between $w = -1$ and $w = -0.5$ is about 4% while that between $w = -1$ and $w = -0.8$ is about 0.9%. For the highly non-linear region, we estimate the difference by combining this perturbative approach with N-body simulation executed for our previous paper. From this, we can expect the difference to be more enhanced in the low-z region, which suggests that the non-Gaussianity of the density distribution potentially plays an important role for extracting the information of dark energy.

1. Introduction

According to recent observations for type Ia Supernovae [1], the expansion of the Universe is accelerating. Combining measurements of Cosmic Microwave Background Radiation [2] and recent galaxy redshift survey [3], researchers have concluded the Universe is almost flat, and we are forced to recognize the existence of a cosmological constant, or a kind of dark energy whose value is almost the same order of magnitude as the present density of the Universe [4]. From the viewpoint of particle physics, however, it is quite difficult to explain such a tiny value that is 14 orders of magnitude smaller than the electroweak scale. The failure of theory to explain the present value of the cosmological constant is known as the "cosmological constant problem" [5].

In order to avoid this problem, many dark energy models have been proposed from various contexts. Roughly speaking, they can be classified into two approaches: one is to modify the gravitational law and the other is to introduce some exotic form of energy. For familiar examples, $1/R$ gravity models [6] and braneworld models [7] belong to the former, while quintessence models [8, 9], k-essence models [10] and phantom models [11] belong to the latter.

Regardless of the attempts mentioned above, it is fair to say that a satisfactory explanation for dark energy has not yet been obtained. On the other hand, from the phenomenological viewpoint, it is important to constrain the effective equation of state of the dark energy w by observations. Even though we have obtained the constraint as $w < -0.78$ (95 % confidence limit assuming $w \geq -1$) [4] by combining WMAP data with other astronomical data, in order to pin down w, it is necessary to use other independent observational tools.

For this purpose, we consider the statistical properties of the large-scale structure of the Universe based on the probability distribution function (PDF) of the cosmological density fluctuations. In the standard picture, the PDF of the primordial density fluctuations originated from quantum fluctuations that were stretched to large comoving scales during the inflation phase and are assumed to be a random Gaussian. It is, however, well known that even though PDF remains Gaussian as long as the density fluctuation is in the linear regime, it significantly deviates from the initial Gaussian shape once the non-linear stage is reached.

Regarding the shape of the PDF, it has been shown that in standard cold dark matter (SCDM) models, the PDF is fairly approximated by lognormal distribution in a weakly non-linear regime [12], while the lognormal PDF does not fit well in a highly non-linear regime [13]. After that, Plionis *et al.* [14] and Borgani *et al.* [15] analyzed the non-Gaussianity of the cluster distribution for several dark matter models including the low-density flat cold dark matter (ΛCDM) model by which the difference of non-Gaussianity between SCDM and ΛCDM models was clearly shown. (The comparison among them by N-body simulations for the quasi-nonlinear stage was also done in [16].) In this paper, furthermore, we intend to investigate the dark energy model dependence for the non-Gaussianity of the PDF, in which we consider the simple constant-w dark

energy models with $w = -0.5, -0.8, -1.0, -1.2$.

In order to analyze the w-dependence of the non-Gaussianity of the PDF, we follow a semi-analytic approach based on Lagrangian linear perturbation theory for the evolution of the density fluctuation. This description provides relatively accurate values even in a quasi-linear regime in the structure formation scenario [12, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. (Especially, for the case of the dust fluid, this approximation is called the Zel'dovich approximation (ZA)). Especially, in many suitable situations, it was shown that the Lagrangian approximation describes the evolution of density fluctuation better than the Eulerian approximation [29, 30, 31, 32].

Another aim of this paper is the extension of our previous paper [33] by including the dependence of the cosmic expansion of the background. This paper is organized as follows. In Sec. 2, we briefly present the evolution equations for the background fluid including the dark energy information. We then also present the perturbation equations based on Lagrangian linear perturbation theory (in Sec. 3) and statistical quantities to investigate non-Gaussianity (in Sec. 4). In Sec. 5 we provide the results and discuss the validity of our perturbative approach. Section 6 is devoted to conclusions.

2. Background

Here we briefly introduce the evolution equations for the background including the dark energy information. If we assume a homogeneous and isotropic Universe, the cosmic expansion law is described by the Friedmann equations and the energy conservation equation.

$$H^2 = \frac{8\pi G}{3}\rho - \frac{\mathcal{K}}{a^2}, \tag{1}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \tag{2}$$

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{3}$$

where a is a scale factor of the Universe, $H = \dot{a}/a$ is its Hubble parameter, $\mathcal{K}$ is a curvature constant, and p and ρ are the total pressure and total energy of matter fields.

As for matter fields, since we are interested in a stage much later than the radiation-matter equality, we consider matter fluid and dark energy, i.e., $\rho = \rho_m + \rho_{DE}$, where ρ_m and ρ_{DE} are the matter fluid energy densities matter fluid and dark energy, respectively. In order to obtain the evolution of the energy densities for each component, we must specify the properties of dark energy which are strongly dependent on dark energy models.

Even though there are some dark energy models in which dark energy couples explicitly to ordinal matter [9], dark energy interacts with ordinal matter only gravitationally in many models. We consider the case in which the energy conservation of each component holds independently. For these situations, since the pressure of the matter is negligible, its energy density evolves as usual:

$$\rho_m \propto a^{-3}. \tag{4}$$

As for the evolution of dark energy, we can use the fact that it is possible to model dark energy as fluid in many dark energy models [5]. For this case, the dynamical properties of dark energy are determined through its effective equation of state w:

$$p_{DE} = w(\rho_{DE})\rho_{DE}, \tag{5}$$

where, in general, w is an analytic function of the energy density, or equivalently, the scale factor. In principle, the evolution of energy density is determined by integrating the energy conservation equation:

$$\rho_{DE}(a) = \rho_{i,DE} e^{-3\int_{a_i}^{a} d\ln a[w(a)+1]}, \tag{6}$$

where $\rho_{i,DE}$ is an integration constant. The condition for the acceleration of the cosmic expansion is given as

$$w < -\frac{1}{3}, \tag{7}$$

where, if we are not to violate the strong energy condition, we should further impose

$$-1 \leq w. \tag{8}$$

However, since it is technically difficult to specify the time dependence of w by observations in near future, we concentrate on the case for constant w, in which case dark energy density evolves as

$$\rho_{DE} \propto a^{-3(w+1)}. \tag{9}$$

Assuming flat spacetime ($\mathcal{K} = 0$) and using Eqs. (4) and (9), we can rewrite the constraint Eq. (1) as

$$H^2 = H_0^2 \left[\Omega_{m0} a^{-3} + \Omega_{DE0} a^{-3(w+1)} \right], \tag{10}$$

where we have defined the density parameters for the matter and the dark energy:

$$\Omega_{(m,\ DE)} \equiv \frac{8\pi G}{3H^2} \rho_{(m,\ DE)}. \tag{11}$$

The subscripts 0 denote the corresponding quantities are evaluated at present.

Under this constraint, we solve the Friedmann equation (Eq. (2)).

3. Perturbation

Next, we introduce the evolution equations for the density fluctuation of matter. Since we are interested in a scale much smaller than the horizon scale and non-relativistic matter fluid, we apply a Newtonian treatment. Notice that after this, ρ denotes the mass density, even though we used the same character as in the previous section. Here, we start with comoving coordinates along the cosmic expansion characterized by the solution $a(t)$ of Eq. (10):

$$\boldsymbol{r} = a(t)\boldsymbol{x}, \tag{12}$$

where $\boldsymbol{r}$ and $\boldsymbol{x}$ are physical coordinates and comoving coordinates, respectively.

In the comoving coordinates, the basic equations for cosmological fluid are described by the continuity equation, the Euler equation and the Poisson equation:

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla_x \cdot \{\boldsymbol{v}(1+\delta)\} = 0\,, \tag{13}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + \frac{1}{a}(\boldsymbol{v}\cdot\nabla_x)\boldsymbol{v} + \frac{\dot{a}}{a}\boldsymbol{v} = \frac{1}{a}\tilde{\boldsymbol{g}}\,, \tag{14}$$

$$\nabla_x \cdot \tilde{\boldsymbol{g}} = -4\pi G\rho_b a\delta\,, \tag{15}$$

where $\delta \equiv \frac{\rho-\rho_b}{\rho_b}$ is the density fluctuation, ρ_b is background density of the matter, $\boldsymbol{v}$ is the peculiar velocity, $\tilde{\boldsymbol{g}} = -\frac{1}{a}\nabla_x\Phi$ is the peculiar acceleration and Φ is the gravitational potential. Since we note non-relativistic fluid as matter, we can ignore the pressure and omit the suffix m.

In this paper, for the perturbation, we adopt the Lagrangian picture rather than the Eulerian picture since we can extract the quasi-nonliniear nature of the structure formation even if we consider only the linear perturbation [29, 30, 31]. For this purpose, it is necessary to define the comoving Lagrangian coordinates $\boldsymbol{q}$ in terms of the comoving Eulerian coordinates $\boldsymbol{x}$ as:

$$\boldsymbol{q} = \boldsymbol{x} + \boldsymbol{s}(\boldsymbol{x},t)\,, \tag{16}$$

where $\boldsymbol{s}$ is the displacement vector denoting the deviation from homogeneous distribution. While in Eulerian perturbation theory, the density fluctuation δ is regarded as a perturbative quantity; in Lagrangian perturbation theory, the displacement vector $\boldsymbol{s}$ is regarded as a perturbative quantity. $\boldsymbol{s}$ can be decomposed to the longitudinal and the transverse modes:

$$\boldsymbol{s} = \boldsymbol{s}^L + \boldsymbol{s}^T\,, \tag{17}$$

$$\nabla \times \boldsymbol{s}^L = \boldsymbol{0}\,, \tag{18}$$

$$\nabla \cdot \boldsymbol{s}^T = 0\,, \tag{19}$$

where ∇ means the Lagrangian spacial derivative.

In Lagrangian coordinates, since we can solve the continuous Eq. (13) from Eq. (16) exactly, we can obtain the density fluctuation as

$$\delta = J^{-1} - 1\,, \tag{20}$$

where J is the determinant of the Jacobian of the mapping between $\boldsymbol{q}$ and $\boldsymbol{r}$: $\partial\boldsymbol{r}/\partial\boldsymbol{q}$. It is worth noting that in Eq. (20), the density fluctuation is given in a formally exact form, even though we keep only the linear term in this paper.

The peculiar velocity appearing in Eqs. (13) and (14) can also be expressed in terms of the Lagrangian coordinate variable $\boldsymbol{s}$ as

$$\boldsymbol{v} = a\dot{\boldsymbol{s}}\,. \tag{21}$$

The basic equations we shall solve are the linearized version of Eqs. (13), (14) and (15) in which δ and $\boldsymbol{v}$ are expressed with $\boldsymbol{s}$ from Eqs. (20) and (21). The solution of the

Lagrangian perturbation can be separated into a time-dependent part and a position-dependent part as:

$$\boldsymbol{s}^{(1)}(t, \boldsymbol{q}) = D(t)\boldsymbol{S}^{(1)}(\boldsymbol{q}) \,. \tag{22}$$

Since it is well known that there are no growing transverse mode solutions in the linear perturbation, in this paper we consider only the longitudinal mode in which the velocity field is irrotational. From the Kelvin circulation theorem, this is quite natural if it is generated only by the action of gravity.

The evolution equation for first-order solution is written as

$$\ddot{D} + 2\frac{\dot{a}}{a}\dot{D} - 4\pi G\rho_b D = 0 \,, \tag{23}$$

while the spacial component is given by the initial condition. Zel'dovich derived a first-order solution of the longitudinal mode for dust fluid [17] which provides the relation between the density fluctuation and the Lagrangian displacement as

$$\delta(\boldsymbol{q}) = -\nabla \cdot \boldsymbol{s}^{L\ (1)} \,. \tag{24}$$

It is worth noting that even though Eq. (24) is obtained by linear perturbation, the value of δ remains accurate even in the quasi-nonlinear regime.

For the low density flat Universe with dark energy whose equation of state is $w = -1$, i.e.,

$$\Omega_m + \Omega_{DE} = 1, \quad \Omega_{DE} \equiv \frac{\Lambda}{3H^2} = \text{const.} \,, \quad \frac{8\pi G}{3}\rho_{DE} = \Lambda \,, \tag{25}$$

the growing mode first-order solution can be expressed with the analytic function.

$$D(t) = \frac{h}{2}B_{1/h^2}\left(\frac{5}{6}, \frac{2}{3}\right) \,, \tag{26}$$

$$h = \frac{H(t)}{\sqrt{\Lambda/3}} \,, \tag{27}$$

where B_{1/h^2} is an incomplete Beta function:

$$B_z(\mu, \nu) \equiv \int_0^z p^{\mu-1}(1-p)^{\nu-1}\mathrm{d}p \,. \tag{28}$$

For other dark energy models, we shall solve Eq. (23) numerically.

In order to avoid the divergence of the density fluctuation in the limit of large k, however, just for a technical reason, it is necessary to consider the density field $\rho(\boldsymbol{x}; R)$ at the position $\boldsymbol{x}$ smoothed over the scale R, which is related to the unsmoothed density field $\rho(\boldsymbol{x})$ as

$$\rho(\boldsymbol{x}; R) = \int d^3\boldsymbol{y}W(|\boldsymbol{x} - \boldsymbol{y}|; R)\rho(\boldsymbol{y})$$
$$= \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\tilde{W}(kR)\tilde{\rho}(\boldsymbol{k})e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \,, \tag{29}$$

where W denotes the window function and $\tilde{W}$ and $\tilde{\rho}$ represent the Fourier transforms of the corresponding quantities. In this paper, we adopt the top-hat window function,

$$\tilde{W} = \frac{3(\sin x - x \cos x)}{x^3} \,. \tag{30}$$

Then, the density fluctuation $\delta(\boldsymbol{x}; R)$ at the position $\boldsymbol{x}$ smoothed over the scale R can be constructed in terms of $\rho(\boldsymbol{x}; R)$. For simplicity we use δ to denote $\delta(\boldsymbol{x}; R)$ unless otherwise stated.

4. Non-Gaussianity of the density fluctuation

In order to analyze the statistics, we introduce a one-point probability distribution of the density fluctuation field $P(\delta)$ (PDF of the density perturbation) which denotes the probability of obtaining the value δ. If δ is a random Gaussian field, the PDF of the density perturbation is determined as

$$P(\delta) = \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-\delta^2/2\sigma^2},$$

where $\sigma \equiv \langle (\delta - \langle \delta \rangle)^2 \rangle$ and $\langle \ \rangle$ denotes the spacial average.

If the PDF deviates from Gaussian distribution, in order to specify it, it is necessary to introduce the following higher-order statistical quantities [12, 34, 35]:

$$\text{skewness} : \gamma = \left\langle \left(\frac{\delta - \langle \delta \rangle}{\sigma} \right)^3 \right\rangle \,,$$

$$\text{kurtosis} : \eta = \left\langle \left(\frac{\delta - \langle \delta \rangle}{\sigma} \right)^4 \right\rangle - 3 \,,$$

which mean the display asymmetry and non-Gaussian degree of "peakiness," respectively.

In Eulerian perturbation theory, the Gaussianity of the PDF completely conserves if we start with Gaussian distribution, because the density fluctuation is described by the product of the time and the spacial component and the spacial component never changes as long as we keep only linear terms. Once nonlinear terms are considered, however, its PDF deviates from the initial Gaussian shape because of the strong nonlinear mode coupling and the nonlocality of the gravitational dynamics. On the other hand, since in Lagrangian perturbation theory, the quasi-nonlinear information in the sense of the Eulerian picture can be extracted even by linear perturbation, we can expect to obtain nontrivial information for the skewness and the kurtosis after $z \sim 5$. See Fig. 1.

5. Numerical results

In this section, after mentioning the conditions we impose, we present our results.

We consider here the Gaussian density field generated by COSMICS [36], and we set the value of the density fluctuation and the peculiar velocity to be those obtained by
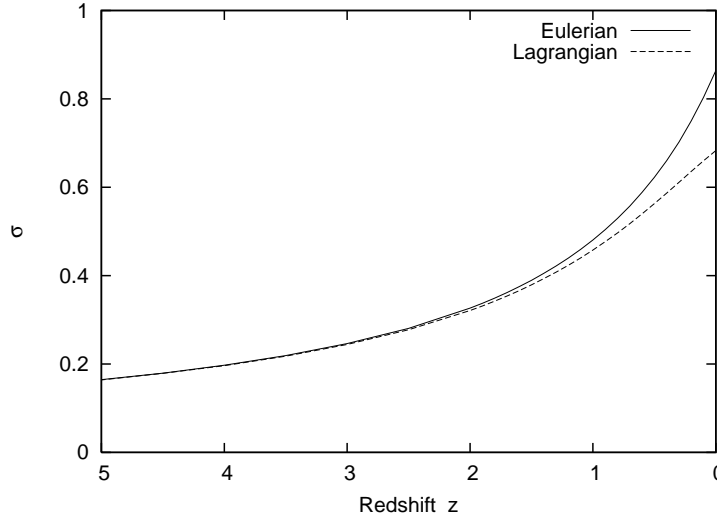
Figure 1. The dispersion of the density fluctuation in both Eulerian and Lagrangian linear perturbation for the model with $w = -1$. Here we choose the smoothing scale $R = 8h^{-1}\mathrm{Mpc}$ for the top-hat window function. After $z \sim 5$, the difference of the dispersion appears between Eulerian and Lagrangian in which Eulerian linear perturbation no longer provides accurate values. Therefore we can expect to obtain the nontrivial information for the skewness and the kurtosis in this quasi-nonlinear region from Lagrangian perturbation as long as it provides accurate values.

Lagrangian linear perturbation with the time evolution of the background to be $a \propto t^{2/3}$. This is reasonable because at this time, the dark energy did not dominate the CDM. Accordingly, both the skewness and the kurtosis are less than 10^{-2} at initial ($z \sim 30$).

For the computation of the Lagrangian perturbation, we set the parameters as follows:

$$\text{Number of grids}: N = 128^3\,,$$
$$\text{Box size} \qquad : L = 128h^{-1}\mathrm{Mpc} \ \ (\text{at } a = 1)\,.$$

Then we impose a periodic boundary condition.

The cosmological parameters at the present time ($a = 1$) are given by [4]

$$\Omega_m \ \ = 0.27\,, \tag{31}$$
$$\Omega_{DE} = 0.73\,, \tag{32}$$
$$H_0 \ \ = 71 \ [\mathrm{km/s/Mpc}]\,, \tag{33}$$
$$\sigma_8 \ \ = 0.84\,. \tag{34}$$

For the dark energy models, we set several equations of state.

$$p = w\rho, \ \ (w = -0.5, -0.8, -1, -1.2)\,, \tag{35}$$

where $w = -1$ corresponds to the cosmological constant. Even though $w = -0.5$ has already been excluded, we calculate this just for comparison. In the case of $w = -1.2$, as mentioned before, the strong energy condition is violated.

As for the smoothing scale R for the top-hat window function, we set $8h^{-1}\mathrm{Mpc}$ in the comoving Eulerian coordinates at the present time ($a = 1$). Because we apply Lagrangian linear perturbation, we must consider the validity of the perturbation. As we will show later, it is reasonable that we set the smoothing scale $R = 8h^{-1}\mathrm{Mpc}$ if we are to discuss the structure formation with the perturbation.

Based on δ calculated from these conditions, we compute the statistical quantities, i.e., the dispersion, the skewness and the kurtosis at 51 time slices from $z = 5$ to $z = 0$ whose time intervals are given as $\Delta z = 0.1$.

Before presenting the results, we must remind ourselves that since these calculations are based on Lagrangian linear perturbation theory, we can trust the results until caustics are produced, i.e., when σ becomes ~ 1.

Figure 2 shows the time evolution of the dispersion of the density fluctuation for several equations of state. The growth of the dispersion monotonously continues until $z = 0$. Because we choose a relatively large value for the smoothing length, the Lagrangian linear perturbation still has physical meaning. The tendency of structure formation can be discussed even if structure formation can no longer be fully described with linear approximation.

We can identify the difference of the dispersion among several values of w. As the value of w becomes larger (approaches to 0), the dispersion becomes smaller. This can be explained as follows: For the model with larger w, dark energy dominates at earlier era, and the expansion of the Universe starts to accelerate at earlier era, while it can be shown that the growth of the density fluctuation is smaller in the accelerating stage than in the matter-dominant stage.

Here we consider high-z region, i.e., $z \geq 2$. In our previous paper [33], we compared the evolution of the dispersion between N-body simulation and Lagrangian linear perturbation. There, the difference of the dispersion, the skewness, and the kurtosis between N-body simulation and the Lagrangian linear perturbation stayed small until $\sigma \simeq 0.3$. From the past analyses, the Lagrangian linear perturbation describes structure formation rather well until $z \geq 2$, where the dispersion is $\sigma \simeq 0.3$.

Withrespect to non-Gaussianity, Figs. 3 and 4 show the evolution of the skewness and the kurtosis, respectively. Like the dispersion, the growth of the skewness and the kurtosis continues monotonously.

For the reason mentioned above, our computations seem to be reliable until $z \simeq 2$. For the case where $w = -0.5$, the value of both the skewness and the kurtosis is obviously less than that for the cases where $w = -1.2$, $w = -1.0$ and $w = -0.8$ (about 4% and about 8%, respectively). On the other hand, the differences of both the skewness and the kurtosis among $w = -1.2$, $w = -1$ and $w = -0.8$ are less than 2% at $z = 2$. Even though these are very small values, they are of almost the same order as the relative ratio of the dispersions. Therefore it can be expected that non-Gaussianity can play a potentially important role in distinguishing the value of w from the observation of high-z galaxies. At that stage, the fitting functions for $\gamma_w(z)$, $\eta_w(z)$ for the quasi-nonlinear stage are useful. Since it seems that the growth of the dispersion and the non-Gaussianity are
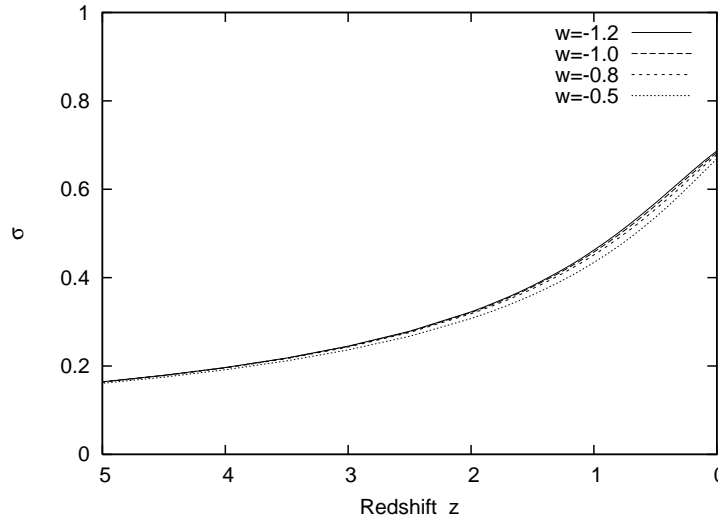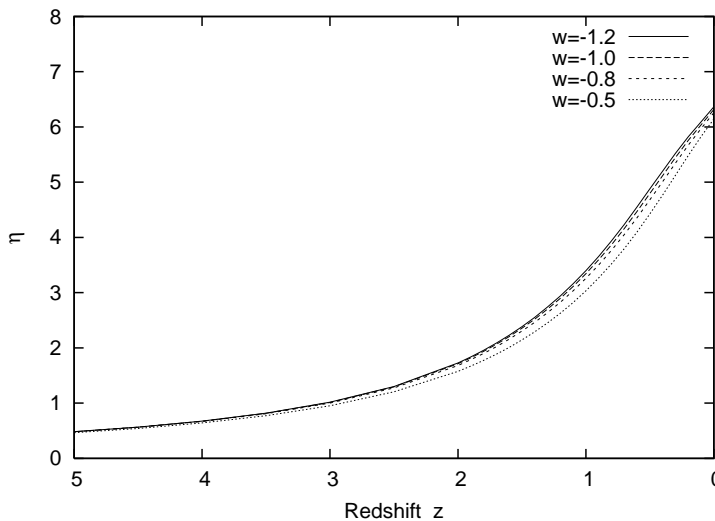
Figure 2. The dispersion of the density fluctuation in several dark energy models. The growth of the dispersion continues monotonously. The difference of the dispersion between $w = -1$ and $w = -0.8$ is about 0.9% at $z = 2$ until which we regard Lagrangian linear perturbation as valid. In the same way, the difference of the dispersion between $w = -1$ and $w = -1.2$ is about 0.4% at $z = 2$.
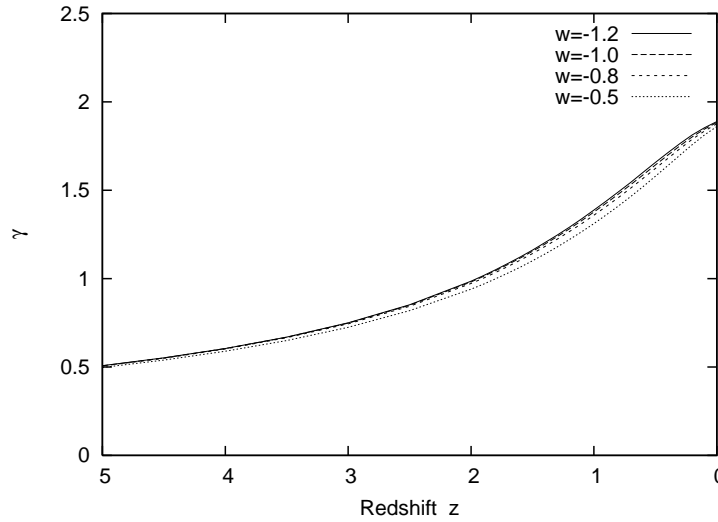
Figure 3. The skewness of the density fluctuation in several dark energy models. The growth of the skewness continues monotonously. Although we can easily distinguish the case where $w = -0.5$ from the results, the difference of the skewness between $w = -1$ and $w = -0.8$ is about 0.9% at $z = 2$ until which we regard Lagrangian linear perturbation as valid. In the same way, the difference of the skewness between $w = -1$ and $w = -1.2$ is about 0.4% at $z = 2$.

related, it is worth trying to find the functions based on our results.

Finally, if we want to know precise information for non-Gaussianity after $z \sim 2$, we must consider nonlinear dynamics by such as N-body simulations. Even though we do not compute them in this paper, judging from Fig.1 in our previous paper, [33], in

Figure 4. The kurtosis of the density fluctuation in several dark energy models. The growth of the kurtosis monotonously continues. As well as the distinction of the skewness, the difference of the kurtosis between $w = -1$ and $w = -0.8$ is about 1.8% at $z = 2$ until which we regard the Lagrangian linear perturbation is valid. In the same way, the difference of the kurtosis between $w = -1$ and $w = -1.2$ is about 0.9% at $z = 2$.

which we compare the results of Lagrangian perturbation and N-body simulation in the Einstein-de Sitter Universe, we can expect the actual non-Gaussianity is strengthened by a order of magnitude. If this happens also for accelerating Universes, it becomes easier to distinguish w from the observations of the near galaxies, which is worth investigating.

6. Summary

To clarify the nature of the dark energy is, without doubt, one of the most important tasks in modern cosmology. From the phenomenological viewpoint, it is important to constrain the effective equation of state of dark energy w by observations. Even though we have obtained the constraint as $w < -0.78$ (95 % confidence limit assuming $w \geq -1$) [4] by combining WMAP data with other astronomical data, to pin down w, it is necessary to use other independent observational tools.

For this purpose, we consider the non-Gaussianity of the density fluctuation which is generated by the non-linear dynamics. In this paper, we follow a semi-analytic approach based on Lagrangian linear perturbation theory for the evolution of the density fluctuation. In this theory, we can extract the quasi-nonlinear feature of the density fluctuation even though we keep only linear terms. In terms of the density fluctuation we obtain, we compute the skewness and the kurtosis which are statistical quantities denoting the degree of non-Gaussianity as well as the dispersion, which is the only parameter for Gaussian distribution. By considering several constant-w dark energy models, we present the w dependence of non-Gaussianity. Because of the validity of the

perturbative approach, we regard the results until $z \simeq 2$ as reliable.

According to our calculation, at fixed time (z), the dispersion becomes smaller as w becomes larger (approaches to 0). This is because the larger w is, the earlier the time at which the expansion of the Universe starts to accelerate and the growth of the density fluctuation is smaller than in the matter-dominant Universe.

The skewness and the kurtosis also become smaller as w becomes larger (approaches to 0), which suggests they have some correlation with the dispersion. The difference of non-Gaussianity between $w = -1$ and $w = -0.5$ is obvious (about 4%) for $2 < z < 5$, while it is hard for w to distinguish -1.2, -1 or -0.8. The differences of both the skewness and the kurtosis between $w = -1$ and $w = -0.8$ as well as $w = -1.2$ and $w = -1.0$ are quite small (about 0.9%), while the correspondent differences of the dispersion are the almost same as these. To specify the value of w, the fitting functions for $\gamma_w(z)$, $\eta_w(z)$ for the quasi-nonlinear stage which can be constructed from our results are useful.

For regions nearer than $z = 2$, it is necessary to pick up non-linear information for obtaining the degree of non-Gaussianity. Even though we have not computed these in this paper, from our previous results comparing N-body simulation with Lagrangian perturbation theory, it can be expected that the actual value of non-Gaussianity becomes larger than the value obtained by Lagrangian linear perturbation. For this case, it becomes easier to distinguish the value of w from the observation of near galaxies, which is worth continuing to consider.

It is necessary to mention the possibility of examining our results. Recently, several galaxy redshift survey projects have been progressing [37, 38, 39]. In these projects, many galaxies within a region ($z < 0.3$ for 2dF, $z < 0.5$ for SDSS, and $0.7 < z < 1.4$ for DEEP2) have been observed. These projects show the latest distribution of galaxies, which form strongly nonlinear structure. For the next generation of spectroscopic surveys, many high-z galaxies will be observed. For example, the Cosmic Inflation Probe (CIP) project [40] is planned to detect objects between $3 < z < 6.5$. Another project, the Kilo-Aperture Optical Spectrograph (KAOS), has also been proposed [41]. One of the primary scientific purposes of KAOS is the determination of the equation of state of dark energy by objects until $z < 3$. Another purpose of KAOS is to observe the growth of structure. The projects leaders propose that the linear growth factor of matter density fluctuations can also provide sensitivity to dark energy properties.

The skewness and the kurtosis of the distribution of galaxies in the SDSS Early Data Release had been computed [42]. The results suggested the SDSS imaging data can enable us to determine the skewness and the kurtosis up to 1% and less than 10%. Therefore in future projects, we can expect to determine the skewness and the kurtosis with high accuracy. From our results, the kurtosis of the density fluctuation seems more sensitive for w than the dispersion of the fluctuation. In other words, although the dispersion reflects the linear growth factor, the kurtosis shows the nature of nonlinear growth in addition to the linear growth. Therefore, structual non-Gaussianity of would show more detailed information on the nature of dark energy. It is worth noting that

for such a high-z region, the results obtained in this paper based on our perturbative approach will play an important role.

Finally, it is worth noting the primordial non-Gaussianity which is generated by inflation. From WMAP observations, a limit for the non-linear coupling parameter has been established [43] which does not deny the existence of non-Gaussianity in the primordial density fluctuation. Even though we cannot disentangle them generally, we can calculate and compare non-Gaussianity of large-scale structure from both Gaussian and non-Gaussian initial conditions. This requires further investigation, and we hope to report results in a separate publication.

Acknowledgments

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